Tempoly
A game designed to learn polynomial operations

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Abstract—In this paper we present a mobile game designed specifically to support the learning of the arithmetic polynomial operations. We describe the characteristics of the game and we state and discuss the reasons behind the choices made in the development of the game.

Keywords—mobile games; mobile learning; game design; mathematical education

I. INTRODUCTION

Learning mathematics presents many challenges. The language is abstract, each new piece of knowledge builds on previous knowledge that the student must master, and a lot of practice is needed to interiorize new mathematical concepts. Once the students start to lose interest in learning this subject, it becomes more and more difficult to keep up with all the demands. It is therefore of extreme importance to keep the students interested in learning mathematics. Conversely, students generally show a great interest in videogames, in particular mobile games, and they get a lot of practice on them. The engagement that students show when they are playing videogames is of vital importance for the learning and consolidation of mathematics concepts.

Mobile learning has several pedagogical advantages (Kukulska-Hulme, 2014, Berge, & Muilenberg, 2013, Crompton, 2013, Parsons, 2007, Traxler, 2007, Carroll et al., 2002, Naismith et al., 2004, Attewell, 2005, 2008, McFarlane et al., 2002, Gee, 2003, 2008, Gros, 2002, Klopfer, 2008, Molenet, 2010, Squire, 2011). These studies show that students have easy access to mobile devices and they can use them anytime outside school. Students are therefore more autonomous in their process of learning. Furthermore, materials can be customized, that is, adapted to each particular student and their current learning progress. Students can also interact more through their devices, be more participative in the learning activities as well as more motivated.

We are interested in studying in which way mobile games can contribute to learn mathematics by students, namely in secondary education. In this work, we focused on the subject of polynomial arithmetic operations, which is part of the national curriculum (Ministério da Educação, 2013). In this paper we describe the work developed, namely the design process of a mobile game about polynomial operations and the motivation for some of the aspects considered in this development.

The paper is organized as follows. In section II, we describe the process of the game design. In particular, we give a brief account of the results of a preliminary survey on the game preferences of students and the motivations for these preferences. In section III, we present the game that was developed. We describe the mechanics of the game and its characteristics, namely from the pedagogical point of view. We focus on the mathematical aspects embedded in the game. In section IV, we illustrate some of the learning principles that we incorporated into the game. For this, we highlight the game elements designed to portray them. In section V, we make some final remarks and propose future work to deepen this study.

II. THE GAME DESIGN

In the initial stage of this work, we developed a survey to identify the preferences of the students concerning mobile games. This survey was answered by 298 students. With this survey we identified the games most played by the students, in different devices. Some of these results are presented in Table 1.

TABLE II - THE GAMES PLAYED MOST BY THE STUDENTS

Device	Laptop	Handheld console	Tablet	Smartphone
Most played games	The Sims	FIFA	Bad Piggies	Hill Climb Racing
	Minecraft	Pro Evolution Soccer	FIFA	Grand Theft Auto
	Pro Evolution Soccer	Grand Theft Auto	Stardolls	Jetpack Joyride
	Grand Theft Auto	Call of Duty	Subway Surfers	Fastball
	Crossfire	Little Big Planet	Jetpack Joyride	Fruit Ninja

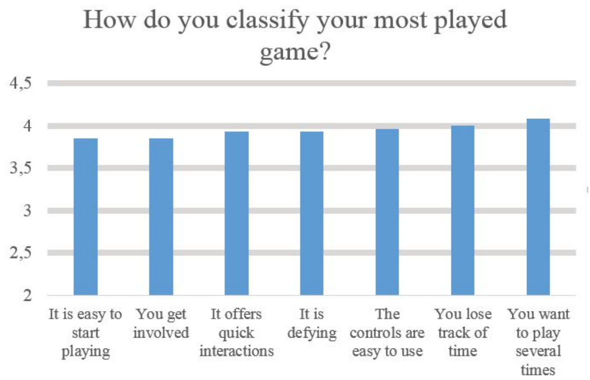

Fig. 1. The main characteristics of the games played most by the students

The survey also intended to identify the characteristics of these games that the students most valued. One of the conclusions obtained from the survey was that the students prefer games with simple and quick interactions. The survey also showed that the students like to play games that are defying.

In addition, we concluded that the games played most by the students reveal characteristics that promote the flow state (Csikszentmihalyi, 1992) of the player. This flow state, which is essential for learning, is characterized by a profound engagement of the player, the feeling that the player has control over the actions needed to finish the activity, and immediate feedback from the game.

We then analyzed the favorite games of the students, in order to understand the learning principles contained in them. Gee (2003) proposed a list of 36 learning principles that are present in videogames. Gee proposes that these learning principles are useful not only in the realm of videogames, but also have applications to learning activities in school. We accepted this proposition and we have incorporated these principles into the design of the game, as described in section IV.

The game development had into account the specific characteristics of mobile devices, as well as the pedagogical dimension of the activity. During the process of the game development, we conducted usability tests with a small number of students. The results of these tests led to some improvements that we incorporated into the game. In particular some suggestions made by the students contributed to the enhancement of the game.

III. THE GAME

The main mathematical subject addressed by the game, named "*Tempoly*" is the four elementary operations on polynomials (addition, subtraction, multiplication and division). Therefore, the game is targeted mainly at the students of the secondary education, since they have in their curriculum the study of these four operations. However, the game can also be used by students from earlier grades, in particular, students of the 3rd cycle of basic education, since they have already an introduction to these operations (namely addition, subtraction and multiplication). Furthermore, we implemented a creative mode in *Tempoly*, which we describe in section IV. The teacher can use this mode to create levels adapted to his/her students, the game can even be used earlier, at the first levels of education.

The purpose of the game is to use some of the elementary operations on a given set of up to four polynomials, to obtain a given result. A popular traditional Chinese game, named the 24-game, which is very popular also in Portugal, is based on a similar idea (Tong et al., 2013), but with some important differences. In the traditional 24-game, the expected result is always 24, and the operations are made with integers from 1 to 10, or, in some versions, from 1 to 13. In *Tempoly* the operations are made with polynomials. In the first levels, however, the degree of these polynomials is zero, so that, in these levels, operations are made only with numbers. Furthermore, in the 24-game, the number of operations used is always three. Conversely, in our game, the number of operations can be whatever number the player desires, without any limit.

The game can be classified as a puzzle game, since the player has to solve several challenges, of increasing complexity. The earlier levels need less operations to be solved and they involve polynomials of smaller degrees. In fact, as mentioned before, the very first levels use only polynomials of degree 0, that is, only numbers. To complete the game, there is a total of 250 challenges, although the player may want to try to solve some extra challenges. All these challenges demand that the player makes use of his problem-solving abilities and critical thinking in mathematics.

The game was developed for the Android operating system, which is very familiar to the students. The manipulation of the polynomials and the operators, and the combination of them, is done by touch. The player has to drag these pieces and combine them, in appropriate ways, to obtain the desired result. There is a large area where the player can execute these operations and store partial results that appear to be useful in future operations (Fig. 2).

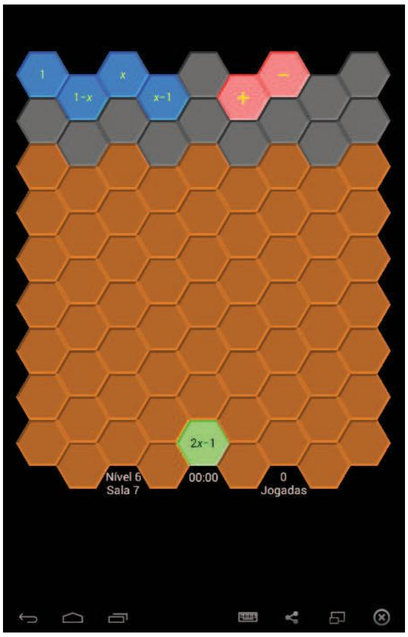

Fig. 2. The board of the game *Tempoly*.

Although the levels were designed for it to be possible to solve in a small number of moves (less than 10), the usability tests showed that it is frequent that the players use a lot more moves (sometimes more than 100) to get to the solution. This large number of moves corresponds to the several attempts that the player sometimes makes, combining the polynomials in different ways. It is also a result of a strategy some players use, of breaking the problem into easier sub-problems, and solving one at a time. For example, to obtain the result $x^3 + 1$, some players first try to obtain the polynomial x^3 and keep this partial result somewhere on the screen. Then they try to obtain 1, and finally they add these partial results.

The name of the game, "*Tempoly*", comes from the words *temple* and *polynomial*, since the graphical elements of the game are themed on a temple. The challenges of the game correspond to rooms of the temple, and solving a challenge corresponds to opening a door, that is providing the right "*key*". There are three areas of the temple, using different materials, namely wood, stone and metal. The player needs to open a certain number of doors in an area to get access to the next area.

In the main menu of *Tempoly*, several actions are proposed to the player. The game lets the player choose a profile, or create a new one, select some options concerning the sounds of the game, check the medals already won and those remaining, see a tutorial, create a new level and, of course, play the game (Fig 3).

The tutorial is very simple, and it describes briefly the game mechanics. The usability tests showed that this tutorial is generally not used by the players. Instead, the first levels of the game are simple enough (since they only involve numbers) to let the players know what they have to do.

If the player wishes to play the game, he can either play one of the predefined levels, or play a level created by another user or even himself. The gameplay is the same in either case, and the player is informed how long (in seconds) did it take to solve the puzzle, and how many moves he has made.

If the player wants to play a challenge that he has solved before, he is informed of his best result, both in terms of time spent and number of movements executed. The player may now try to beat his own best result. The purpose of solving an already solved challenge is to obtain better solutions to the problem. This corresponds to the principle of Polya (1945) of reviewing a solved problem in order to produce a better solution.

Fig. 3. The main menu of the game.

In each level, the player uses one or more of the available polynomials and operations, to obtain the proposed result. There are generally many different solutions, and the player can use any one of them. For example, in Fig. 2, the player could obtain the result $2x - 1$ with one operation: $x + (x - 1)$, two operations: $(x - 1) + (x - 1) + 1$, or propose a more complex calculation with many more operations. In this process, the player can never "lose" the level, as the game allows him to keep trying to obtain a solution with the polynomials he has already obtained.

The calculations are done in real time and they are presented to the player through an animation that inserts the polynomials into the two upper corners of the operator and makes the result of the operation come out of it, accompanied by a sound of a turning crank. It is not formally explained how these operations are carried through, but since the game starts with very simple cases, and then slowly more complex cases appear, the player can gradually form a mental picture of how they work.

The objects are presented in hexagonal boxes and the building area is also made out of hexagonal cells. This design comes from the fact that three operations, namely the addition, the subtraction and the multiplication have only one result (which comes out of the operator from below), but the division has two results: the quotient and the remainder (which comes out of the operator respectively from the lower left and the lower right).

The challenges are grouped in 25 levels, and each level has ten different challenges, totalizing 250 different challenges. The first ten levels are in the wood area, the next ten are in the stone area and the final five are in the metal area. Each group of five levels corresponds to a different polynomial degree: in the first five levels there are only numbers, in the next five the player combines polynomials of degrees up to one, in the next five up to two, and so on, until the final five levels, which involve operations with polynomials of degree up to four. The challenges in each group of five levels are sorted by their difficulty and the number of operations required to complete the challenge.

When the player solves a challenge, he gets access to the next level. The game then proposes a challenge from that level, and the player will gradually try to solve increasingly difficult problems. In some occasions, when the player solves a challenge, he gets a medal corresponding to his achievement. There is a total of 20 medals, which reward different aspects of the gameplay, such as speed and perseverance. Once a level gets unlocked, it stays forever unlocked and the player can play as many times as he/she wants on an unlocked level.

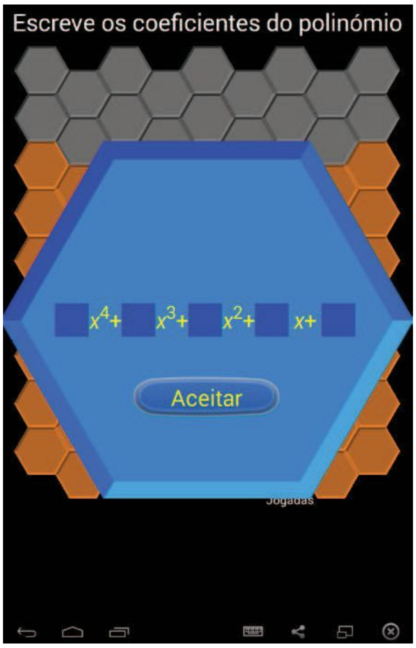

Fig. 4. The game's creative mode

The game has a creative mode, that lets the player build new challenges. These challenges can then be played by himself or by other players. To build a new challenge, the player needs to define the polynomials that will be available in that challenge. He does this by choosing the polynomial coefficients (Fig. 4). The player also needs to select the available operations. He then combines these newly defined elements as if he was playing the game. When he obtains the result he intends to propose, he can select it and the challenge is automatically saved.

IV. LEARNING PRINCIPLES

Several aspects of the game design, mentioned in the previous section, are rooted in the learning principles present in videogames (Gee, 2003). By using these learning principles, we intend to develop the students' problem solving abilities, and to promote their critical thinking. We will now describe, with more detail, how some of these learning principles were translated into characteristics of the game and how they were incorporated into the game design.

Active, Critical Learning Principle – All elements of the game are set up in a way that the player needs to actively reflect on the challenges presented to him. He needs to search for a solution involving the combination of several of these elements. Furthermore, the player can analyze the problem and decompose it in several sub-problems. The player doesn't limit himself to passively reading about or even watching a presentation of some concepts about polynomials.

Design Principle – As mentioned before, three of the operations have only one result, while the division requires two results, the quotient and the remainder. The hexagonal shape of the pieces allows to game to present this important difference to the player, and the understanding of this design element is essential for the learning of the subject of polynomial operations.

"Psychosocial Moratorium" Principle – The game never penalizes the player for proposing a wrong solution. To open a door, the player needs to provide the correct key, and the door does not open without it. However, while the player does not get it open, he can attempt different approaches, or alternatively, he can keep trying to build on what he has already accomplished. Therefore, the player feels that every partial result that he has already obtained is not wrong, but is instead another step in the path to obtain a solution. The design of the game, which presents each challenge as a puzzle instead of an exercise, as it is usually seen in scholar contexts, also promotes the feeling of the player actions having no consequences in the real world, allowing him to try multiple approaches without fear of repercussions.

Amplification of Input Principle – The game multiplies greatly the input given by the player. As the player combines the pieces that are available on the screen, he can, with very simple and intuitive movements, produce quite complex calculations. The player can therefore experiment a much greater number of alternatives that he could not without using the game.

Achievement Principle – The game *Tempoly* rewards the achievements of the player, throughout all the game. This process begins with the first solutions proposed by the player, and it continues until the player has completed all 250 challenges. Furthermore, there are rewards for creating new challenges and for improving the player's own results. The game signalizes the moments when the medals are won, and the player can see at any time what achievements are valued by the game (Fig. 5).

Practice Principle – The player has lots of opportunities to practice the four polynomial operations, due to the large number of increasingly difficult challenges. The game design has been set up to promote the engagement of the player, signalizing his progress in several ways. The game rewards the player with medals, unlocks more complex levels, and shows the player how many challenges he has completed in each level.

Fig. 5. The list of medals, with the indication of those already won.

"Regime of Competence" Principle – As the player progresses in the game, the complexity slowly but steadily increases. As a consequence, at each stage, the challenge presented to the player, although it may appear difficult, it also seems possible to solve. Furthermore, since the game allows the creation of new challenges, the teacher can also use specific challenges adapted to the abilities and the knowledge of his students.

Probing Principle – At each level of the game, the player has the opportunity to formulate a hypothesis on how to solve the challenge, and to try this hypothesis by explicitly combining some of the elements displayed on the screen. If this attempt does not solve the problem, the result, or results, of the operation are still displayed. The player can then use these results to transform the original problem to a simpler problem, or he can try a completely new approach. The challenge solving process is therefore a constant cycle of analysis, reflection and testing.

Multiple Routes Principle – Each challenge has several solutions, and the player can present any one of them to solve it. The player is allowed to discover new solutions, even if they were not predicted by the creator of the challenge. The solutions are not limited in terms of the number of operations used. The same challenge can be played several times, so that the player can try to find simpler solutions.

Multimodal Principle – The game design combines elements of different modalities to create the meaning of the objects. For example, a polynomial operation is executed when an operator is placed between two polynomials. The execution of the operation is a combination of text elements that describe what it being calculated, a graphical animation showing the operands being combined and transformed into the result(s), and the sound a mechanical device. This suggests the meaning of an operation as a machine, or, in mathematical terms, as a function.

Material Intelligence Principle – The operators in the game are intelligent, this is, they carry with them the knowledge on how to perform the respective operations. They also carry some rules concerning polynomials, such as the impossibility of dividing by zero, the notion of equality of polynomials or the irrelevance of the letter chosen for the variable of the polynomial. The game intentionally forces some of this knowledge to be presented to the player. For instance, a simple operation such as adding the polynomials 1 and x may result in the expression $1 + x$ or the expression $x + 1$ (which may look different but correspond to the same polynomial). If an expression is written differently from what is shown in the hexagon containing the solution, but corresponds to the same polynomial, the answer is always accepted. Another example appears when the player decides to replay a specific challenge. In this situation the game may present the same challenge using a different symbol for the variable. The player will nevertheless see the problem as being the same.

Intuitive Knowledge Principle – The player has the opportunity to produce much more calculations that we would do traditionally, using pen and paper. Through this repeated experience with the execution of the polynomial operations presented to him, he/she may gain an intuitive knowledge on how to solve each challenge. The player may get the feeling, just by looking at the available polynomials, of which he must combine, and with which operators, even without explicitly making the corresponding calculations.

Incremental Principle – The game levels are ordered according to their difficulty. The simpler situations, regarding the number of operations necessary and the degrees of the polynomials involved, appear earlier in the game. These simpler situations, in many cases, can be used by the players to solve sub-problems of the more complex situations that appear later. The more complex cases are, in a way, just compositions of very simple cases, and they do not appear to be unapproachable.

Insider Principle – The game *Tempoly* has a creative mode, which allows the player to also be a producer of new content. In this new role, the player must actively think about the meaning of the objects and symbols of the game, with the objective of combining them to produce new objects and therefore creating new challenges in the game.

V. FINAL REMARKS

In this paper we described a game that we have developed recently, for mobile devices, on the subject of polynomial arithmetic operations. This game was developed having in mind the game preferences of the students and Gee's learning principles concerning videogames. The game, named *Tempoly*, was presented to a small number of students, under the scope of a usability test, with the purpose of improving some details of the game.

The final version of the game will soon be used in a study to evaluate the learning benefits of the game and the reactions of the students to the use of the game in the educational context.

We hope that the use of videogames in school contributes to improve student engagement in learning activities. This is an essential condition to give the students a deeper understanding of the mathematical concepts involved in the game and to make them more proficient in using the related mathematical tools.

Acknowledgment

This research project is funded by FEDER through the Programa Operacional Fatores de Competitividade – COMPETE and by National Funds through FCT – Fundação para a Ciência e a Tecnologia under the project PTDC/CPE-CED/1187/2010.

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